

# A shock dynamics approach for analyzing two-dimensional numerical detonation cells in gases

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The reaction zone of a detonation in gases exhibits a three-dimensional, unsteady cellular structure. Thus, the leading shock of self-sustained detonations consists of Mach stems –the cells– with longitudinal shocks that cyclically decelerate from velocities greater than to lesser than the Chapman–Jouguet value  $D_{CJ}$ , with continuously decreasing total curvature. Relationships between the normal shock velocity  $D_n$ , the normal acceleration  $\dot{D}_n$ , and the total curvature  $\kappa$  have been commonly used to analyze the dynamics of these shocks and the physics of the flow that sustains them.

For example, Jackson et al. [1, 2] have analyzed the cell dynamics with  $D_n$ - $\kappa$  and  $D_n$ - $\dot{D}_n$ - $\kappa$  relationships for the detonation in a straight channel. Based on a  $D_n$ - $\kappa$  relationship evaluated from the shock front of a cell, they identified three features [1]: reactive Mach stem, inert blast wave, and the transition between the two. With a  $D_n$ - $\dot{D}_n$ - $\kappa$  relationship evaluated along the normal to the detonation cell front, they observed a surface in the  $D_n$ - $\dot{D}_n$ - $\kappa$  space during a cell cycle [2]. However, their analysis addressed the specific case of a very weakly unstable mixture with a one-step chemistry, a high specific heat ratio  $\gamma \sim 1.6$ , and a single cell in the channel width. By contrast, the present work considers  $D_n$ - $\dot{D}_n$ - $\kappa$  relationships evaluated from two-dimensional simulations, with detailed chemistry and several cells in the channel width [3, 4].

In the main, for the regular hydrogen-based mixture  $2\text{H}_2 + \text{O}_2 + 7\text{Ar}$ , the same three features as in [1, 2] are observed. For the irregular  $2\text{H}_2 + \text{O}_2$  mixture, forward jetting and compression waves from the delayed combustion of unburnt pockets can significantly modify  $\kappa$  and  $\dot{D}_n$ . Further analysis then confirms the shock-ignition criterion expressing boundedness of the induction time [5–7], i.e., a rate of heat production greater than that of adiabatic cooling from volumetric expansion, that shock-flame decoupling in cells depends more on flow unsteadiness ( $\sim \dot{D}_n$ ) than on flow divergence ( $\sim \kappa$ ).

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