Shock-induced breakup of liquid fuel jet: a coupled momentum and energy transfer theory

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1 Introduction

Liquid fuels are beneficial in several aspects of rotating and oblique detonation wave (ODW) as well as in scramjet engine operations [1–4] for their potentially high energy densities and favorable penetration into oxidizer cross-flow. However, the efficiency and power density of these engines can be impacted by the breakup of the liquid fuel jet, vaporization and mixing upon interactions with a shock or in a supersonic cross-flow. The deflection of the liquid jet upon encountering a supersonic gas cross-flow, and the mechanisms of jet breakup and fragmentation are therefore central to the design of these combustors. The jet deflection is related to the momentum transfer between the cross-flow and the liquid jet, while the time and distance required for the jet to break up depends on energy transfer. These processes are fully coupled. A fundamental understanding of the coupled energy and momentum transfer mechanism is needed to model liquid jet breakup in shocks and supersonic cross-flows.

Recent work on shock-induced vaporization of liquid droplets [5] for shock strengths typical in detonations suggests that the breakup process is dictated by the impingement of the post-shock flow on the liquid mass. The impact, whether it is directly on the droplet surface or through the formation of a reflected shock, creates a differential pressure force on the droplet, leading to its rapid flattening due to its inertia. In fact, the flattening of the droplet is a manifestation of non-equilibrium compression heating [6], which is significantly faster than what would have been predicted by the d^2 law and its variants. The findings are in direct contrast to a wealth of literature studies, experimentally [7–9] and theoretically [10, 11], that focused on relatively low-speed flows or weak shocks where liquid surface tension effects are pronounced. Under typical detonation conditions, the liquid fuel mass would quickly attain supercritical fluid state following flow impaction, diminishing the influence of surface tension [5].

In the current work, we use the findings for liquid droplet breakup in a shock [5] to inform a theory for the breakup of a liquid jet in a supersonic cross-flow. The theoretical approach is similar to that developed by Kateris et al. [6] but is now applied in a quasi-two-dimensional setup in which a liquid column breaks up in a supersonic cross-flow. Specifically, momentum transfer from a high-speed cross-flow on a liquid parcel within the fuel jet results in the flattening and displacement of the parcel, leading to the deflection of the liquid jet in a cross-flow. Energy transfer is coupled to momentum transfer and determines the location along the jet streamtube where breakup occurs. *Breakup* is defined as the state in which the liquid fuel parcel has been vaporized and reaches the same temperature and pressure as the post-shock cross-flow.

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2 Theory

We consider the steady injection of a fuel jet crossing a supersonic flow. The fuel jet is simultaneously deflected and flattened as it encounters a post-shock flow of speed U and density ρ_g as illustrated in Fig. 1. The supersonic cross-flow imparts momentum on the jet, deflecting it and transferring energy that is partitioned between (i) acceleration of fuel parcels to the cross-flow velocity U and (ii) rise of the internal kinetic energy of fuel parcels in such a way that they eventually vaporize and then attain the pressure and temperature of the post shock gas. Here, the breakup point, as indicated in Fig. 1, differs from the historical definition of spray breakup. As discussed earlier, we define here the breakup point to be a state at which the fuel is vaporized and reaches the temperature and pressure same as the post-shock gas. At the breakup point, the centerline of the jet makes an angle θ_v with respect to the direction of the flow. The high-speed cross-flow interacts with the fuel jet, transferring momentum and energy into the liquid jet in such a way that the fluid attains supercritical fluid state quickly, thus leading to negligible viscous and surface tension effects [5] following the initial interactions. In other words, we consider here the regime of large Weber and Reynolds numbers only.

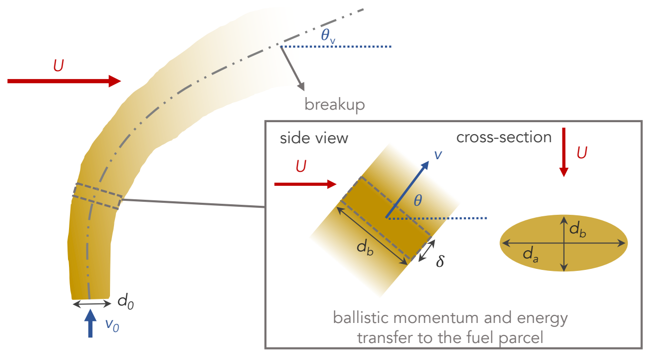

Figure 1: Schematic of the injection of liquid fuel at velocity v_0 through a circular nozzle of diameter d_0 into a supersonic cross-flow of speed U. The inset shows a fuel parcel moving with a velocity $\mathbf{v}$ at an angle θ to the flow direction. The parcel has thickness δ and an elliptical cross-section with effective major and minor axes of d_a and d_b respectively. The jet centerline, which tracks the center of mass motion of the fuel parcel, is indicated by the dotted lines.

The fuel parcel is assumed to be an elliptical cylinder with thickness δ and a cross-section with time-varying major and minor axes d_a and d_b respectively, that describes the deformation of the jet cross-section (Fig. 1). The jet cross-section is initially circular with $d_a = d_b = d_0$ and then flattens along the streamline so that $d_a > d_0$ and $d_a > d_b$. The flattening of the fuel parcel occurs in low-speed jet-cross-flow interactions [12,13] and it is expected to be more significant in high-speed flows. Mass conservation for the liquid parcel of density ρ_l requires that $\rho_{l,0}\pi d_0^2\delta_0/4 = \rho_l \pi d_a d_b \delta/4$ and mass continuity for a control volume drawn around the parcel results in $\rho_{l,0} v_0 d_0^2 = \rho_l v d_a d_b$. These two forms of conservation can be combined to yield

$$\frac{\delta}{\delta_0} = \frac{v}{v_0}. \tag{1}$$

The rates of momentum and energy transfer to the jet from the compressed gas of density ρ_g and temperature T_g convecting at a speed of U can be determined by considering the impact of the flow. Under steady conditions, the static pressure of the gas surrounding the jet is uniform and the fuel jet itself is assumed to have constant internal pressure. Therefore, the dominant force acting on the fuel parcel comes from the impact of gas molecules on the exposed jet surface. The exposed fluid parcel surface has a projected area $d_a\delta$ and only the leading surface is attacked by the cross-flow, as shown in the inset in Fig. 1. Compared to the pressure forcing on the leading surface, the back surface of the liquid jet experiences negligible momentum transfer from the flow due to significant flow velocity and pressure differences. Clearly, this differential front-back momentum transfer results in jet deflection. Assuming full momentum accommodation of gas molecules to the fuel parcel [6] and considering the momentum flux from the cross-flow impact and Eq. (1), we find the local centripetal force F_{cent} to be

$$F_{\text{cent}} = \frac{\pi}{4}\rho_{l,0} d_0^2 \delta_0 v_0 \dot{\theta} = \rho_g d_a \delta U^2 \sin^2\theta, \tag{2}$$

where $\dot{\theta}$ is the local angular velocity and the local radius of curvature is $v/\dot{\theta}$. Given the steady-state assumption, we can parameterize the trajectory of the fuel parcel with time t such that $\theta(t)$ marks the fuel parcel's location within the jet streamtube. Then, the following semi-empirical model describes the flattening of the jet cross-section [6]:

$$d_a = d_0 e^{\zeta t/\tau}, \tag{3}$$

where τ is a characteristic time-scale for momentum transfer. The shape response factor ζ determines the rate of flattening of the fluid parcel, which depends on the liquid properties and surrounding pressure [6], and it needs to be determined by experiments. With this model, we may integrate Eq. (2) to yield a solution for θ along the jet centerline:

$$\tan\theta = \frac{v_0}{U}\frac{\zeta}{e^{\zeta t/\tau} - 1}. \tag{4}$$

The jet shape captures the radial momentum transfer and can be used to obtain the value ζ. The characteristic time-scale τ can be deduced by determining the drag acting on the fluid parcel from the cross-flow. The drag force F_{drag} on the parcel along the direction of its velocity is

$$F_{\text{drag}} = \rho_g d_a \delta U \sin\theta (U\cos\theta - v), \tag{5}$$

and the equation of motion of the parcel is

$$\frac{\pi}{4}\rho_0 d_0^2 \delta_0 \frac{dv}{dt} = \rho_g d_a \delta U \sin\theta (U\cos\theta - v), \tag{6}$$

allowing us to define

$$\tau = \frac{\pi}{4}\frac{\rho_{l,0}}{\rho_g}\frac{d_0}{U} \tag{7}$$

as the momentum relaxation time. We can integrate Eq. (6) to obtain

$$v = \frac{U v_0}{U\sin\theta + v_0\cos\theta}, \tag{8}$$

which satisfies the conditions that $v = v_0$ when $\theta = \pi/2$ at the jet origin and $v = U$ when $\theta = 0$ when the fuel jet has accommodated completely to the momentum of the cross-flow.

To determine the location of breakup θ_{v}, we consider the energy transfer between the cross-flow gas and the fuel parcel. The relative kinetic energy flux to the exposed area $d_a\delta$ is

$$\dot{E}_g'' = \frac{1}{2}\rho_g U \sin\theta [(U - v\cos\theta)^2 + v^2\sin^2\theta]. \tag{9}$$

At breakup, i.e., $\theta(t_{\text{v}}) = \theta_{\text{v}}$ we obtain

$$\int_0^{t_{\text{v}}} \left(\dot{E}_g'' d_a \delta\right) \, dt = \rho_{l,0}\frac{\pi d_0^2}{4}\delta_0 \Delta h. \tag{10}$$

If the post-shock gas has a temperature of T_g and the liquid is injected at a temperature of $T_{0,l}$, Δh determines the total amount of enthalpy that must be transferred to the parcel so that it reaches the same temperature as the gas and is given by

$$\Delta h = \left[h(T_{\text{vap}}) - h(T_{0,l})\right]_{\text{sens},l} + \Delta h_{\text{vap}} + \left[h(T_g) - h(T_{\text{vap}})\right]_{\text{sens},g}, \tag{11}$$

where T_{vap} is the saturation temperature at the prevailing pressure and Δh_{vap} is the latent heat of vaporization. $\left[h(T_{\text{vap}}) - h(T_{0,l})\right]_{\text{sens},l}$ and $\left[h(T_g) - h(T_{\text{vap}})\right]_{\text{sens},g}$ are the sensible enthalpy differences in the liquid and gas/supercritical phases of the fuel respectively. Since enthalpy is a state variable, the above formulation applies for both equilibrium and non-equilibrium energy transfer [5,6].

The expression for the flux in Eq. (9) can be substituted in the integral in Eq. (10) to yield the condition:

$$\int_{\theta_{\text{v}}}^{\pi/2} \frac{\sin\theta}{(\sin\theta + (v_0/U)\cos\theta)^3}\, d\theta = \frac{2\Delta h U}{v_0(U^2 + v_0^2)}, \tag{12}$$

which can be simplified by assuming that $v_0/U \ll 1$. Evaluating the expression, we obtain

$$\tan\theta_{\text{v}} = \frac{v_0}{U}\frac{(U^2 + v_0^2)/2}{\Delta h}. \tag{13}$$

Equivalently we can determine the time t_{v} that the fuel parcel undergoes breakup after being released from the injector to be

$$t_{\text{v}} = \frac{\tau}{\zeta}\log\left[1 + \zeta\frac{\Delta h}{(U^2 + v_0^2)/2}\right]. \tag{14}$$

3 Results and Discussion

The theory is applied to predict the profile of an *n*-dodecane jet in supersonic cross-flow. Fig. 2a shows the shape of the centerline (determined from Eq. (4)) of an *n*-dodecane jet injected through a circular injector of diameter d_0 at a speed of $v_0 = 30$ m/s and liquid temperature of 263.60 K ($\rho_{l,0} = 771.69$ kg/m^3) into an air cross-flow with $U = 1619.9$ m/s, temperature $T_g = 2759.7$ K and pressure $P = 0.164$ bar. The conditions correspond to the flow behind a M = 7 shock propagating through quiescent air at 263.60 K and 287 Pa. The jet shape in Fig. 2a is strongly dependent on the shape response factor ζ indicating that experimental measurements of the fuel jet profile can yield useful information about the dynamics of the flattening of the jet cross-section. The fuel jet profile for $\zeta = 1000$ is only plotted until the breakup point as defined by the condition in Eq. (13). A larger value of ζ increases the momentum and energy transfer rate as a larger cross-section is exposed to the cross-flow, decreasing both the penetration depth and the horizontal range of the jet centerline.

The results from the theory can also be compared against available experimental data for *n*-dodecane injection in high-speed flows. Fig. 2b compares the theoretical jet edge against the apparent jet edge extracted from schlieren data [9] for *n*-dodecane injection through a circular injector of diameter $d_0 = 0.5$ mm into a cross-flow of speed $U = 1595$ m/s at temperature $T_g = 1200$ K and pressure $P = 1.6$ bar. The fuel velocity at injection v_0 was varied to achieve momentum flux ratios ($J = \rho_{0,l}v_0^2/\rho_g U^2$) of 4.1, 5.2 and 6.8. To compare the theoretical calculation of the jet edge with the experimental jet edge, the local semi-major axis $d_a/2$ of the jet cross-section was added to the centerline evaluated using Eq. (4) (while correcting for the parcel orientation). As a result of the flattening of the jet cross-section, $d_a/2$ would be an overestimate for the offset between the jet centerline and the jet edge. Therefore, the theoretical calculations of the jet edges in Fig. 2b are upper bounds. By applying a constant vertical translation of y_{sh}, the theoretical jet edge was anchored to the apparent injection location. For the

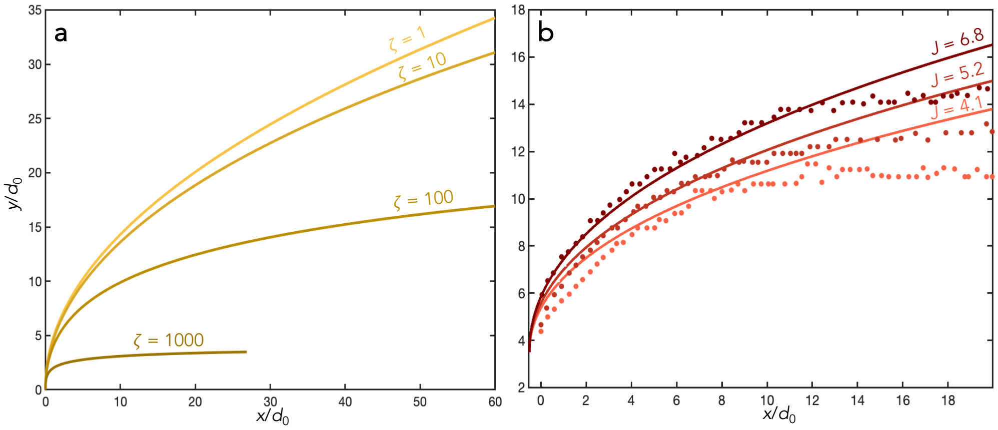

Figure 2: (a) Normalized penetration y/d_0 with respect to normalized horizontal distance x/d_0 for the centerline of a n-dodecane jet ($v_0 = 30$ m/s) injected in a supersonic cross-flow at $U = 1619.9$ m/s and $T_g = 2759.7$ K and pressure of 0.164 bar at varying ζ; (b) comparison between an experimental apparent jet edge [9] and theoretical calculations of n-dodecane injection at varying momentum flux ratios.

comparison in Fig. 2b, we use $y_{\text{sh}} = 3.5\ d_0$ and $\zeta = 3$ across all momentum flux ratios considered. We observe from Fig. 2b that the theory agrees with the observed jet edge for small x/d_0 values but then overshoots the experimental jet edge. The overshoot is most likely because of the inability of schlieren to resolve small fuel clusters shed from the jet. This issue is well-known in the subsonic jet-cross-flow literature [14], and the discrepancy between the apparent and true jet penetration is expected to be even larger for a liquid jet impacted by a supersonic cross-flow.

4 Conclusions

A coupled momentum and energy transfer theory is derived for the breakup of a liquid fuel jet in a supersonic cross-flow. The theory predicts the jet shape and the location of breakup based on an impact-driven momentum and energy transfer mechanism. Supersonic flow impingement on the liquid parcels that constitute the fuel jet induces a differential momentum transfer. This causes the flattening of the jet cross-section. The flattening increases the rates of energy and momentum transfer to the fuel jet from the supersonic cross-flow, leading to the rapid deflection and breakup of the fuel jet. The rate of flattening is described by a shape response factor, which can be obtained from experimental measurements of the jet shape for liquid fuel injected into a supersonic gas cross-flow. The theory can then be used to predict the jet breakup location, which would be useful in the design of combustors for multiphase detonation engines, scramjet engines and other high-speed combustion applications.

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